ELECTRON CONDUCTIVITY OF A THERMALLY IONIZED GAS IN AN ELECTRIC FIELD

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 21–24, 1966

It is well known that the current carriers in a thermally ionized gas vary in composition, but that electrons [1] make the fundamental contribution to the conductivity of the gas, since their mobility is incomparably larger than that of other current-carrying particles. We shall thus be concerned only with electron conductivity. If the gas is under a high pressure in a weak electric field, then in estimating its electrical conductivity by classical means the same concepts are usually employed as those which Drude applied in the theory of metallic conduction. The Drude-Lorentz formula for electrical conductivity was subsequently perfected by Cowling and Chapman who introduced a coefficient to take into account the rate at which the particle interaction forces decrease with distance [2]. For electron Coulomb interaction this coefficient takes the value 0.532 instead of 0.500 as compared with the Drude-Lorentz formula.

For high pressures and low electric field strengths the electron drift velocity in the field is vanishingly small compared with the mean velocity of random motion, and so it is logical to suppose that the electron free time is independent of the drift velocity, and this supposition leads in the end to the conclusion that Ohm's law is applicable to gases at high pressure in very weak fields.

However, we must not overlook the fact that even under the conditions mentioned the conclusion concerning the validity of Ohm's law is only an approximation which becomes less accurate, the lower the gas pressure and the greater the field strength.

In what follows the conductivity of the gas is also determined by Drude's method, but with the refinement that in determining the electron free time the drift velocity of these particles in the field is considered.

Let the concentration of free electrons be known, also their mean effective collision cross section, and consequently the mean free path. Let the distribution of quantities in the gas be spatially isotropic and independent of time. Assuming also that the macroscopic parameters of the state of the gas are given we will find the conductivity σ in accordance with the definition

$$\sigma = en_e v / E. \tag{1}$$

Here e is the electronic charge, n_e is the free electron concentration, v is the mean drift velocity during the free time on an interval λ.

Since all quantities on the right side of (1), with the exception of v, are assumed to be given, it follows that the search for the expression σ reduces to a determination of the mean drift velocity of the electrons. This problem may be solved exactly if the distribution function of electron velocities in a gas situated in an external field is known. This function has, however, been found only in the first approximation [1], which is suitable for weak fields only; it will be shown below that in order to determine σ it is not necessary to know the distribution function, but that it suffices to use kinematic relationships only.

We surround some particle of the gas (figure) with a sphere of radius λ. Clearly, inside this sphere there are $N = 4\pi\lambda^3 n_e/3$ free electrons. Drifting under the influence of the field (in Fig. 1 it is parallel to the x axis), the electrons will be scattered on the particle. After scattering, in general the electrons will move in the field along curved trajectories. We must not lose sight of the fact that the fundamentals modes of collision are electron-ion and electron-molecule interactions. Molecules and ions have large mass and so their mobility in fields which are not very strong does not depend on the field strength. In view of this we may assume that the free motion of electrons, with the stipulation that the gas is isotropic, will be bounded by the surface of a spherical cell with a radius of exactly λ. If we confine ourselves to fields which are not strong, we may assume that the intensity of electron scattering is virtually uniform in all directions.

We will consider the motion of electrons in an element of a spherical sector with aperture angle α. Since the volume of the element is equal to $2\pi\lambda^3/3 \times \times \sin\alpha\, d\alpha$, and under our conditions the scattering intensity is independent of angle, $dN_\alpha = n_0\, 2\pi\lambda^3/3 \times \times \sin\alpha\, d\alpha$ electrons move in the element. The action of the field on these electrons is manifested kinematically in the fact that during the time taken to traverse an interval λ of free path, a drift velocity v_α is added to the component of mean random motion parallel to the field v_x. Geometrically, the effect of this action reduces to rotating the mean velocity V through some angle (figure) about the scattering center. We shall find the drift velocity v_α. To do this we write

$$\lambda \sin(\alpha - \xi) = V \sin\alpha\, \tau_\alpha \qquad \left(\tau_\alpha = v_\alpha \frac{m}{eE}\right) \tag{2}$$
$$\lambda \cos(\alpha - \xi) = (V\cos\alpha + {}^1/_2 v_\alpha)\tau.$$

Here $\lambda \sin(\alpha - \xi)$ is the component of the free path of the electrons inside the element under consideration, normal to the field, $\lambda \cos(\alpha - \xi)$ is the component of the path parallel to the field, τ_α is the free time of the electrons, expressed in terms of the drift velocity and the acceleration which they experience in the field.

Equations (2) may be simplified. This is due to the fact that in weak fields in the absence of vacuum phenomena the drift velocity is several orders smaller than the average thermal velocity. Of course, the expression $\sqrt{\lambda e E/m}$ may be used to estimate the drift velocity. If λ is set equal to 10^{-5} cm in this expression, which corresponds to a pressure of p = 1 mm (in actual conditions p is usually higher), then v_α will be of the order of 10^5 cm/sec, and the mean velocity of random motion of the electrons in these conditions is roughly 10^8 cm/sec, i. e., two or three orders higher than the drift velocity. For large pressures p and small E the difference between V and v_α is clearly large. If this is taken into account, then, as is clear from the figure, the absolute value of the angle ξ must be considered to be small, and so the system of equations (2) transforms to the simpler form

$$\lambda(\sin\alpha - \xi\cos\alpha) = V\sin\alpha\,\tau_\alpha, \qquad \left(\tau_\alpha = \frac{mv_\alpha}{eE}\right).$$
$$\lambda(\cos\alpha + \xi\sin\alpha) = (V\cos\alpha + {}^1/_2 v_\alpha)\tau_\alpha, \tag{3}$$

Solving these equations for v_α, we find

$$v_\alpha = \frac{1}{\cos\alpha}\left(V^2 + \frac{2e\lambda E}{m}\cos\alpha\right)^{1/2} - V. \tag{4}$$

We will now determine the drift velocity of the electrons inside the sphere. It is equal to

$$v = \frac{1}{2N}\int_0^\pi v_\alpha\, dN_\alpha\ . \tag{5}$$

Setting in dN_α and v_α and making the change of variable $\cos\alpha = x$, we obtain a standard integral.

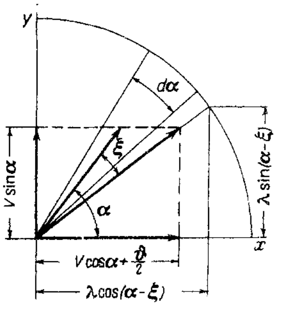
Using [3], we find

$$v = 2\frac{E^*}{\sqrt{V^2+2E^*} + \sqrt{V^2-2E^*}} +$$
$$+\frac{V}{4}\ln\frac{2V - (\sqrt{V^2+2E^*} - \sqrt{V^2-2E^*})}{2V + (\sqrt{V^2+2E^*} - \sqrt{V^2-2E^*})}, \quad \left(E^* = \frac{e\lambda E}{m}\right). \tag{6}$$

Finally, setting (6) in (1), we obtain the refined classical formula for the electrical conductivity

$$\sigma = 2\frac{n_e e^2\lambda}{m}\frac{1}{\sqrt{V^2+2E^*} + \sqrt{V^2-2E^*}} +$$
$$+\frac{n_e eV}{4E}\ln\frac{2V - (\sqrt{V^2+2E^*} - \sqrt{V^2-2E^*})}{2V + (\sqrt{V^2+2E^*} - \sqrt{V^2-2E^*})}\ . \tag{7}$$

It is clear that σ is a complicated function of the field strength E and gas pressure p, since $\lambda = \lambda_0 p$, and, moreover, the degree of complexity becomes even greater if we take into account that n_e, λ and V also depend on E in strong fields. For a more definite estimate of σ from (7) we will confine ourselves to two physically simple situations.



Let the mean energy which the electrons acquire in the field over an interval λ be much less than kT (very weak field), then n_e and λ may be considered as constants, and the velocity of random electron motion, in accordance with Maxwell's law, will be a function of temperature only

$$V = \left(\frac{8kT}{\pi m}\right)^{1/2}. \tag{8}$$

Setting (8) in (7) and letting E in (7) tend to zero, we obtain, as one would expect, the Drude-Lorentz formula

$$\sigma = \frac{1}{2}\frac{n_e e^2\lambda}{m\sqrt{8kT/\pi m}}\ . \tag{9}$$

The case when the mean electron energy in the field is much larger than kT is also important. For a gas at temperature T, of the order of 1000°, for example, and pressure p of the order of 1 mm Hg, this condition will be fulfilled in a field of strength E up to 1 V/cm. In this case, as Druyvesteyn [1] showed, the mean velocity of random electron motion is determined by the expression

$$V = \frac{1}{\varkappa}\left(\frac{2e\lambda_0 E}{mp}\right)^{1/2}, \qquad \varkappa = \left(\frac{4m}{M}\right)^{1/4}. \tag{10}$$

Here λ_0 is the electron free path for unit pressure, for example, for p = 1 mm, M is the ion or molecular mass.

	E/p	u/p	$up(E/p)\pi$
Ne	0.20	2.80	1.26
	0.25	2.50	1.25
	0.30	2.30	1.26
	0.48	2.00	1.36
	0.65	1.70	1.37
	1.00	1.40	1.40
He	0.48	1.43	0.99
	0.65	1.25	1.01
	0.92	1.00	0.96
	1.00	0.94	0.94
Ar	0.10	2.40	0.77
	0.14	2.00	0.76
	0.20	1.60	0.72
	0.30	1.10	0.60

Thus in the fields considered the velocity of random electron motion is a function of the field. If we calculate the velocity of random motion according to Dryuvesteyn for the conditions given above, it turns out to be an order higher than the electron drift velocity. Thus the condition that the angle ξ be small remains valid. At the same time, in a field strength of the order of 1 V/cm we may still neglect inelastic collisions, and so n_e and λ_0 may, as in the first case, be considered independent of the field. Allowing for this, on setting (10) in (17) we obtain

$$p\sigma = \left(\frac{2n_e e^3\lambda_0}{m}\right)^{1/2}\left(\frac{p}{E}\right)^{1/2}\left[\frac{\varkappa}{\sqrt{1+\varkappa^2} + \sqrt{1-\varkappa^2}} + \right.$$
$$\left. + \frac{1}{4}\frac{1}{\varkappa}\ln\frac{2 - \sqrt{1+\varkappa^2} + \sqrt{1-\varkappa^2}}{2 + \sqrt{1+\varkappa^2} - \sqrt{1-\varkappa^2}}\right]. \tag{11}$$

Since $\varkappa$ is of the order of 1/6 or less, expression (11) may be simplified. To do this we expand the factor in square brackets in powers of $\varkappa$ and reject terms of higher than first order

$$p\sigma = \frac{1}{4}\varkappa\left(\frac{2n_e^2e^2\lambda_0}{m}\right)^{1/2}\left(\frac{p}{E}\right)^{1/2}. \qquad (12)$$

The resulting expression coincides with the expression for the electron conductivity of gases obtained by the statistical method [1]. It is clear from (12) that variation of the electrical conductivity depends closely on p/E. The nature of the variation is such that the product $p\sigma\sqrt{E/p}$ should be constant in the physical conditions under consideration.

The table gives values of u/p [cm^2 · mm/V · sec]. Here u is the mobility of free electrons in the gas, a quantity which, in the case being considered, is proportional to σ, E/p [V/cm · mm Hg] and the product $\pi = pu\ (E/p)^{1/2}$ for Ne, He and Ar. We calculated all these values from graphical data given in Ebert's manual [4].

It follows from the table that up to the well-known limit of the argument E/p, $p\sigma\sqrt{E/p}$ is actually close to a constant. We shall compare the result obtained with the Drude-Lorentz formula. The Drude-Lorentz formula assumes that the mobility of the current carriers is independent of the field strength. The mobility is proportional to λ or 1/p. Thus the product u · p should not depend on E/p. It is clear from the table, however, that as E/p increases the magnitude of u · p decreases continuously. Thus for argon, when E/p increases from 0.1 to 0.3 the product u · p decreases to less than half its value. Thus formula (7) describes the experimental data considerably better than the Drude-Lorentz formula.

At large E/p relation (12) is violated. This is physically understandable. At large values of the parameter E/p inelastic collisions become important, as a result of which not only the velocity of the electrons, but also their concentration and free path depend on the field.

In conclusion, we stress once more that when certain conditions are fulfilled equation (7) is suitable for evaluating the electron conductivity of gases in strong fields. If the field is stationary, the gas isotropic and relatively remote from a state of molecular vacuum, then the basic prerequisite that the mean drift velocity be less than the mean velocity of random motion also remains valid in this case. If this is so, then, determining the electrical conductivity with an accuracy to the unknown functions n_e, λ and V, we obtain expression (7).

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28 September 1965 Chelyabinsk